

UWTSD

Module

Task 2

Name: Hassan Ali Raza
Module Code: ACHA7002
Student Number: 2513043
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1 Bisection

A function was created based on equation 1.1 to solve roots using the bisection method

$$x_2 = (x_1 + x_3)/2 \quad \text{Equation 1-1}$$

1.1 $f(x) = \sin(x) \sinh(x)$

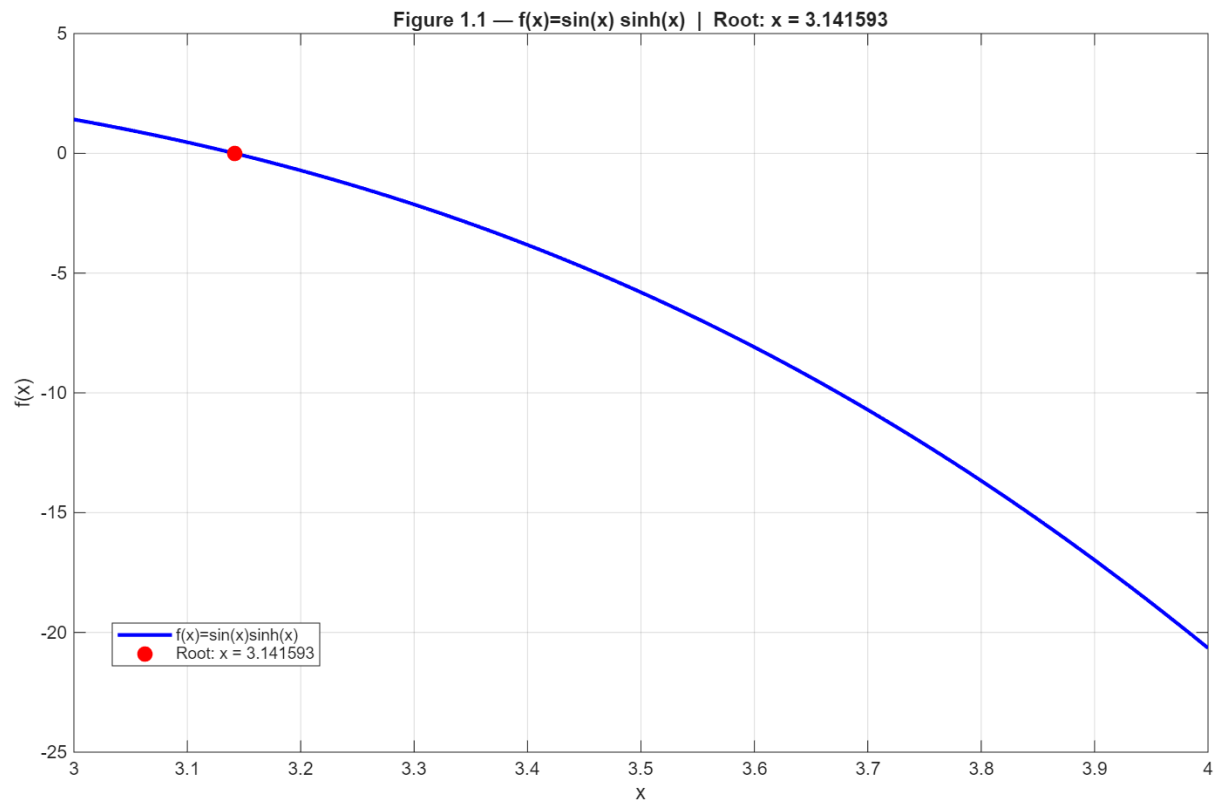


Figure 1-1 this shows the function labelled in the legend and the root (Function 1 root: $x = 3.141593$)

1.2 $f(x) = (R^2 - x^2)^{1/2} - x \tan(x)$

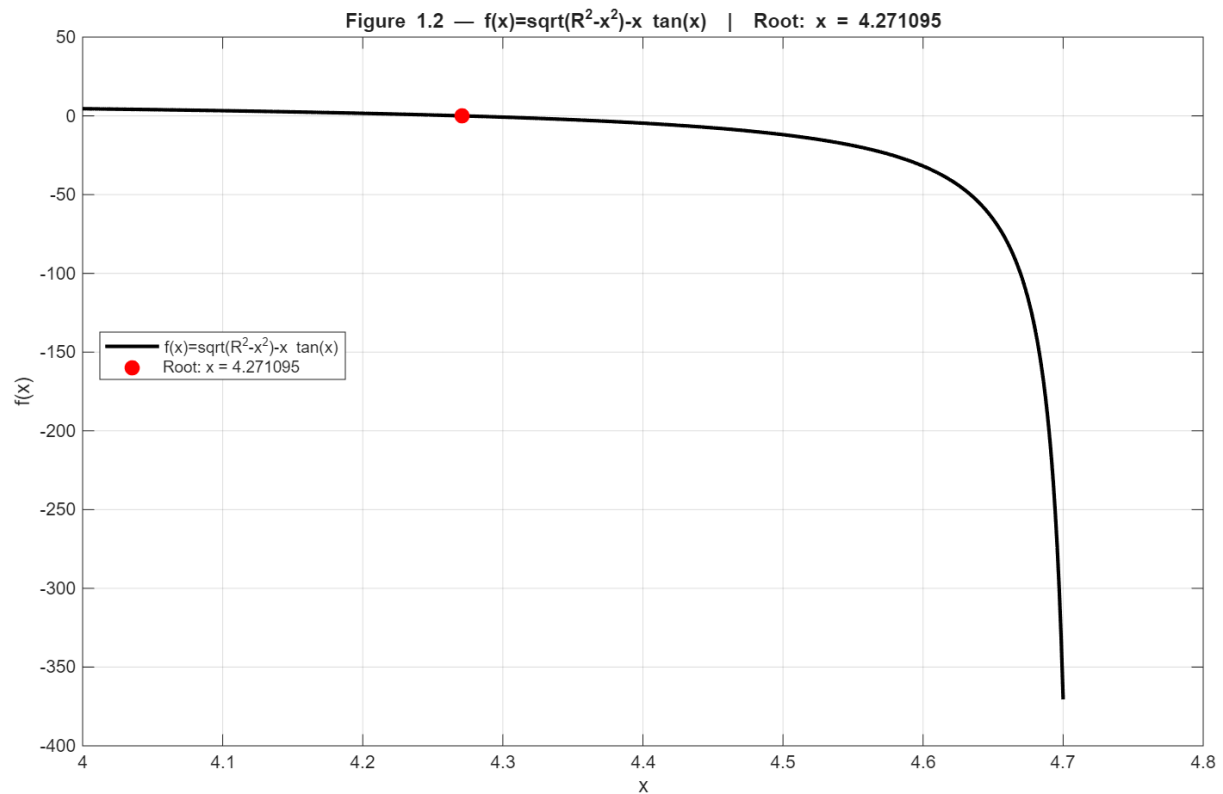


Figure 1-2 this shows the function labelled in the legend and the root (Function 2 root: $x = 4.271095$)

2 Newton's method

A function was made from equation 2.1 to evaluate roots with the newton method to use this method, functions and their derivative functions were created.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{Equation 2-1}$$

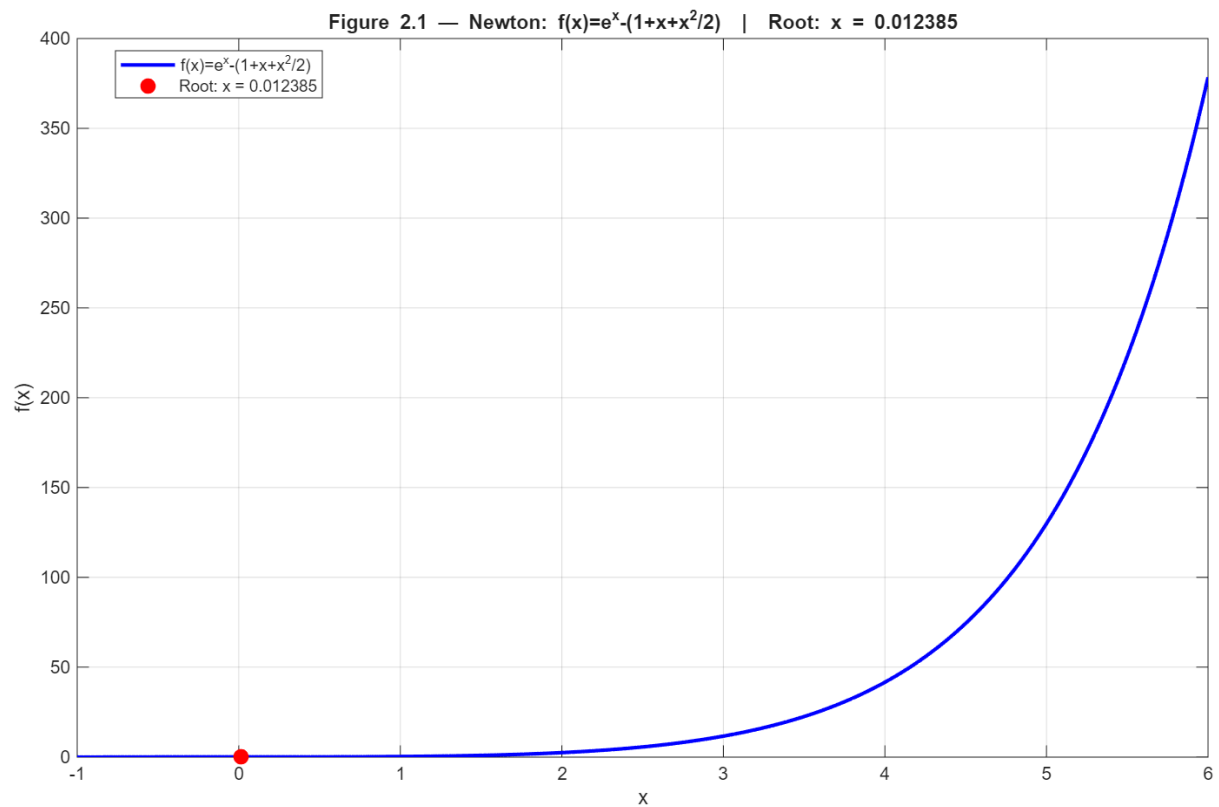
2.1 Newton b $f(x) = e^x - (1 + x + (x^2/2))$ 

Figure 2-1 this shows the function labelled in the legend and the root

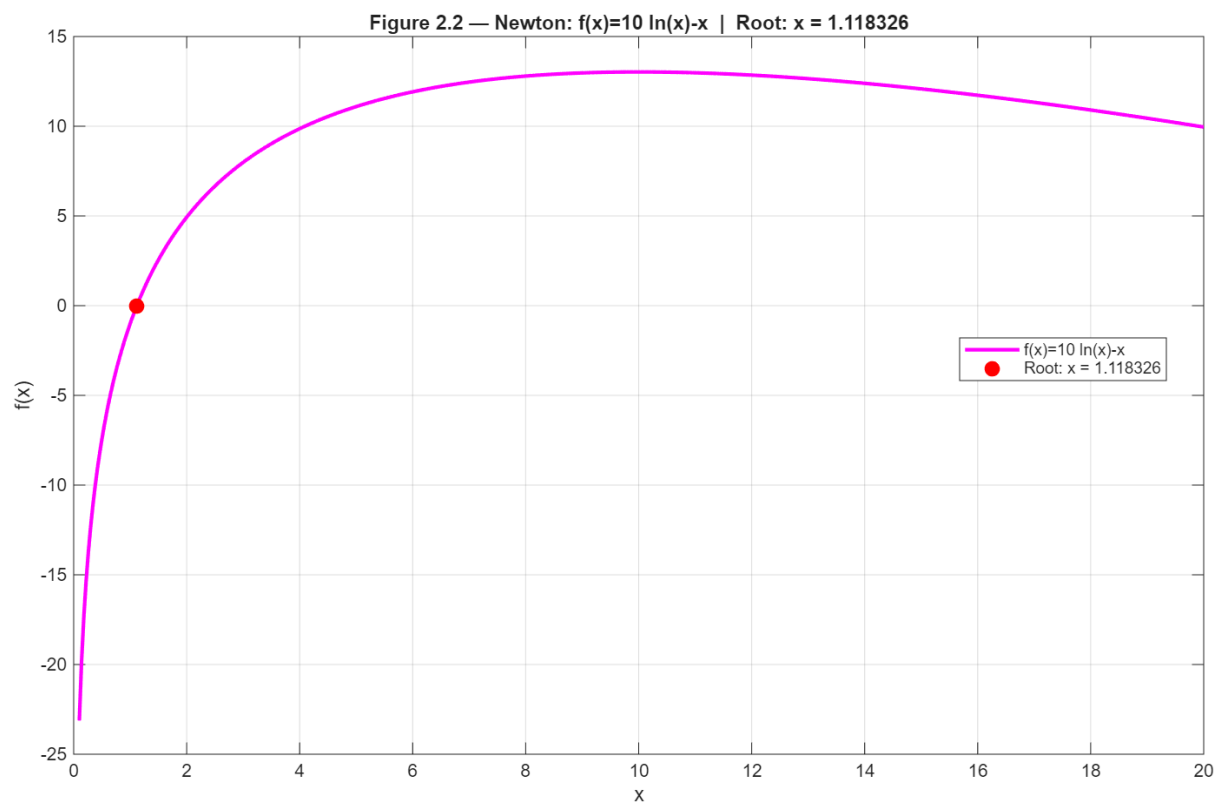
2.2 Newton c $f(x) = 10 \log(x) - x$ 

Figure 2-2 this shows the function labelled in the legend and the root

3 Intersect plot $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

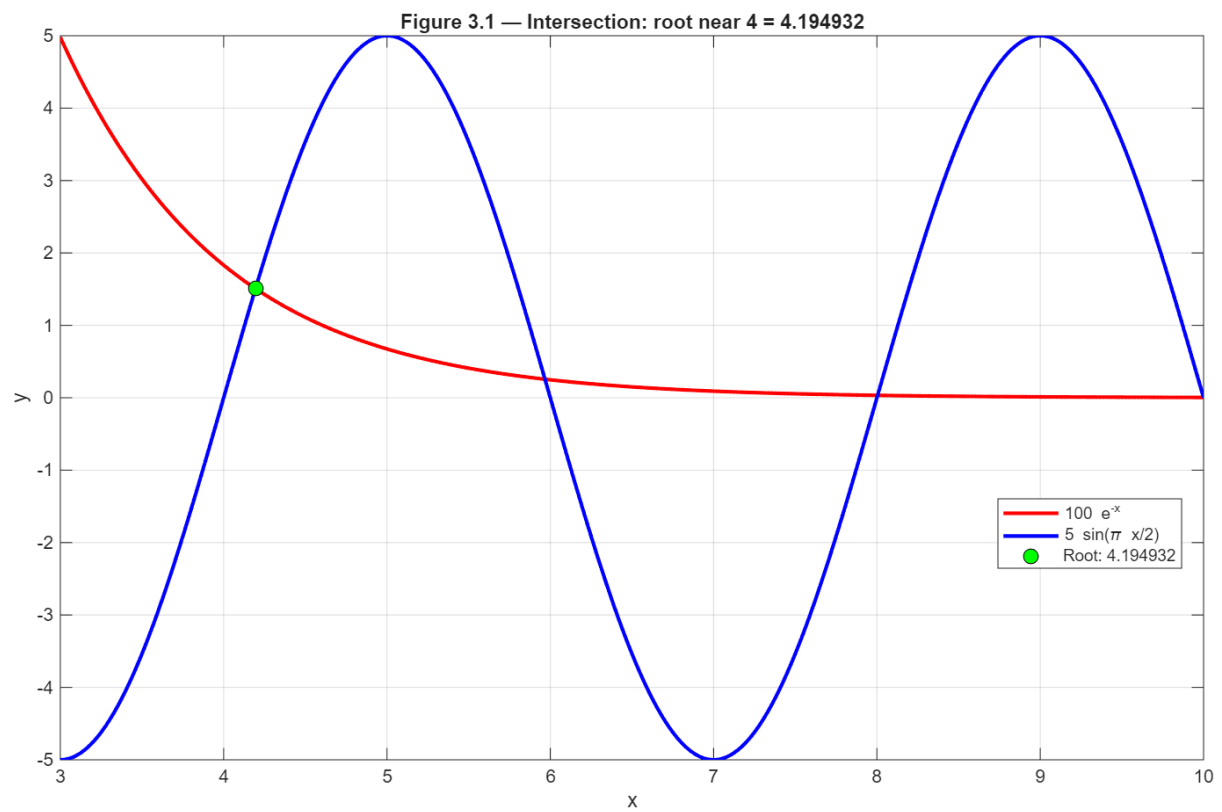


Figure 3-1 plot of function intersection

4 Bracketing

4.1 Bisect $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

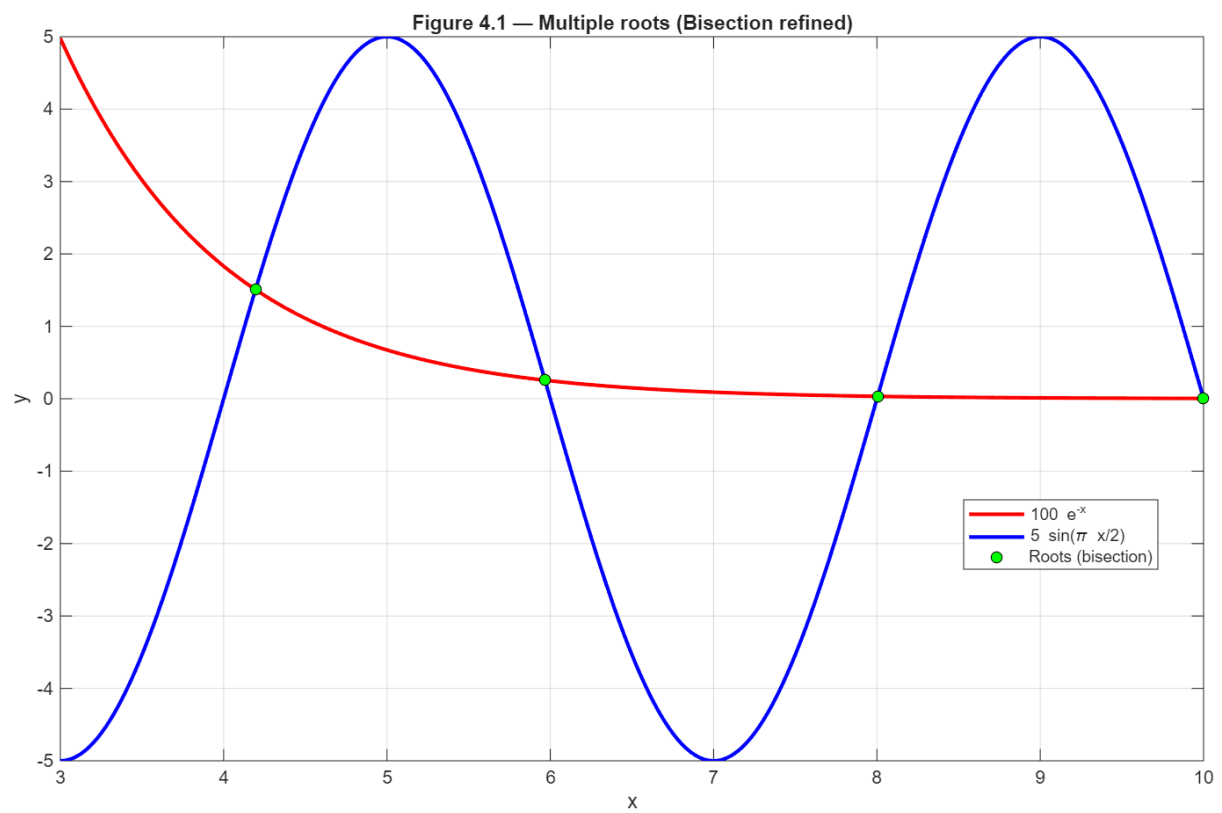


Figure 4-1 multiple roots Bisect for functions shown in legend

4.2 Newton $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

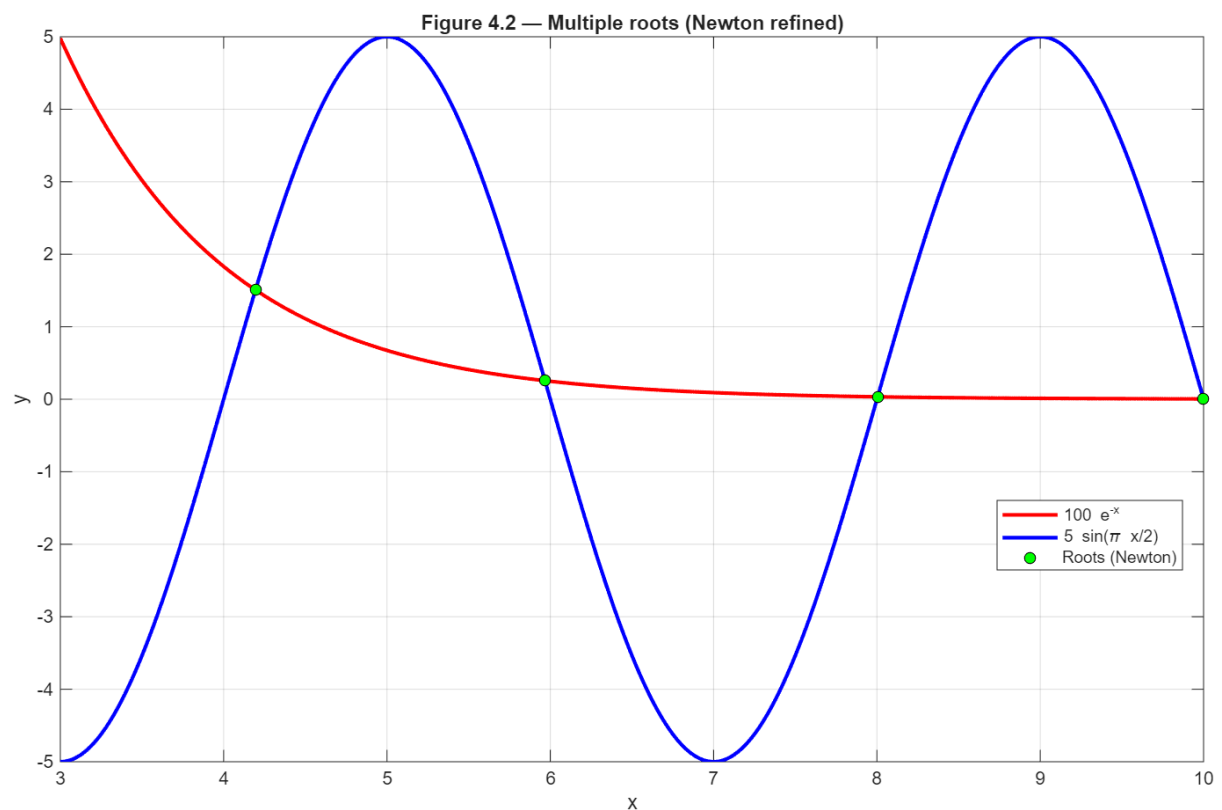


Figure 4-2 multiple roots Newton for functions shown in legend

5 Appendices

5.1 Convergence table Bisection $f(x) = \sin(x) \sinh(x)$

CONVERGENCE TABLE: $f(x) = \sin(x) \sinh(x)$

Interval: [1, 4], tol=1e-6

Iter	a	b	Middle	f(Middle)	Interval Size	Error Bound
1	1.000000	4.000000	2.500000	3.62e+00	3.00e+00	1.50e+00
2	2.500000	4.000000	3.250000	-1.39e+00	1.50e+00	7.50e-01
3	2.500000	3.250000	2.875000	2.33e+00	7.50e-01	3.75e-01
4	2.875000	3.250000	3.062500	8.43e-01	3.75e-01	1.88e-01
5	3.062500	3.250000	3.156250	-1.72e-01	1.88e-01	9.38e-02
6	3.062500	3.156250	3.109375	3.60e-01	9.38e-02	4.69e-02
7	3.109375	3.156250	3.132812	1.01e-01	4.69e-02	2.34e-02

```

      8 | 3.132812 | 3.156250 | 3.144531 | -3.40e-02 | 2.34e-02 |
1.17e-02
      9 | 3.132812 | 3.144531 | 3.138672 | 3.36e-02 | 1.17e-02 |
5.86e-03
     10 | 3.138672 | 3.144531 | 3.141602 | -1.03e-04 | 5.86e-03 |
2.93e-03
     11 | 3.138672 | 3.141602 | 3.140137 | 1.68e-02 | 2.93e-03 |
1.46e-03
     12 | 3.140137 | 3.141602 | 3.140869 | 8.35e-03 | 1.46e-03 |
7.32e-04
     13 | 3.140869 | 3.141602 | 3.141235 | 4.12e-03 | 7.32e-04 |
3.66e-04
     14 | 3.141235 | 3.141602 | 3.141418 | 2.01e-03 | 3.66e-04 |
1.83e-04
     15 | 3.141418 | 3.141602 | 3.141510 | 9.54e-04 | 1.83e-04 |
9.16e-05
     16 | 3.141510 | 3.141602 | 3.141556 | 4.26e-04 | 9.16e-05 |
4.58e-05
     17 | 3.141556 | 3.141602 | 3.141579 | 1.61e-04 | 4.58e-05 |
2.29e-05
     18 | 3.141579 | 3.141602 | 3.141590 | 2.93e-05 | 2.29e-05 |
1.14e-05
     19 | 3.141590 | 3.141602 | 3.141596 | -3.68e-05 | 1.14e-05 |
5.72e-06
     20 | 3.141590 | 3.141596 | 3.141593 | -3.76e-06 | 5.72e-06 |
2.86e-06
     21 | 3.141590 | 3.141593 | 3.141592 | 1.28e-05 | 2.86e-06 |
1.43e-06
     22 | 3.141592 | 3.141593 | 3.141592 | 4.50e-06 | 1.43e-06 |
7.15e-07

```

```

>>> CONVERGED: Root = 3.14159262 after 22 iterations
      f(root) = 3.67e-07, Error Bound = 7.15e-07

```

5.2 Convergence table Bisection $f(x) = (R^2 - x^2)^{1/2} - x \tan(x)$

5.2 CONVERGENCE TABLE: $f(x) = \sqrt{R^2 - x^2} - x \tan(x)$

Interval: [4, 4.7], $R=10$, $\text{tol}=1e-6$

Iter	a	b	Middle	f(Middle)	Interval Size
1	4.000000	4.700000	4.350000	-2.47e+00	7.00e-01
2	4.000000	4.350000	4.175000	2.08e+00	3.50e-01
3	4.175000	4.350000	4.262500	2.20e-01	1.75e-01
4	4.262500	4.350000	4.306250	-9.88e-01	8.75e-02
5	4.262500	4.306250	4.284375	-3.55e-01	4.38e-02
6	4.262500	4.284375	4.273437	-6.12e-02	2.19e-02
7	4.262500	4.273437	4.267969	8.08e-02	1.09e-02
8	4.267969	4.273437	4.270703	1.02e-02	5.47e-03
9	4.270703	4.273437	4.272070	-2.54e-02	2.73e-03
10	4.270703	4.272070	4.271387	-7.58e-03	1.37e-03

11	4.270703	4.271387	4.271045	1.31e-03	6.84e-04
12	4.271045	4.271387	4.271216	-3.13e-03	3.42e-04
13	4.271045	4.271216	4.271130	-9.11e-04	1.71e-04
14	4.271045	4.271130	4.271088	2.00e-04	8.54e-05
15	4.271088	4.271130	4.271109	-3.55e-04	4.27e-05
16	4.271088	4.271109	4.271098	-7.77e-05	2.14e-05
17	4.271088	4.271098	4.271093	6.11e-05	1.07e-05
18	4.271093	4.271098	4.271096	-8.31e-06	5.34e-06
19	4.271093	4.271096	4.271094	2.64e-05	2.67e-06
20	4.271094	4.271096	4.271095	9.05e-06	1.34e-06

5.3 Convergence table Newton b $f(x) = e^x - (1 + x + (x^2/2))$

3 CONVERGENCE TABLE: $f(x) = e^x - (1 + x + x^2/2)$

Initial guess: $x_0=5$, $\text{tol}=1e-6$

Iter	x_n	$f(x_n)$	$f'(x_n)$	dx
1	5.00000000	1.30e+02	1.42e+02	9.12e-01
2	4.08777279	4.62e+01	5.45e+01	8.47e-01
3	3.24102047	1.61e+01	2.13e+01	7.54e-01
4	2.48738115	5.45e+00	8.54e+00	6.38e-01
5	1.84952185	1.80e+00	3.51e+00	5.12e-01
6	1.33718652	5.77e-01	1.47e+00	3.92e-01
7	0.94490680	1.81e-01	6.28e-01	2.89e-01
8	0.65615109	5.59e-02	2.71e-01	2.06e-01
9	0.44988343	1.70e-02	1.18e-01	1.44e-01
10	0.30570599	5.15e-03	5.19e-02	9.93e-02
11	0.20645146	1.55e-03	2.29e-02	6.76e-02
12	0.13883426	4.62e-04	1.01e-02	4.57e-02
13	0.09309648	1.38e-04	4.47e-03	3.08e-02
14	0.06230655	4.09e-05	1.98e-03	2.07e-02
15	0.04164598	1.22e-05	8.79e-04	1.38e-02
16	0.02781230	3.61e-06	3.90e-04	9.25e-03
17	0.01856306	1.07e-06	1.73e-04	6.18e-03
18	0.01238496	3.18e-07	7.70e-05	4.12e-03
19	0.00826090	9.42e-08	3.42e-05	2.75e-03
20	0.00550916	2.79e-08	1.52e-05	1.84e-03

>>> MAX ITERATIONS REACHED: Current $x = 0.00367362$

5.4 Convergence table Newton c $f(x) = 10 \log(x) - x$

5.4 CONVERGENCE TABLE: $f(x) = 10 \log(x) - x$

Initial guess: $x_0=1$, $\text{tol}=1e-6$

Iter	x _n	f(x _n)	f'(x _n)	dx
1	1.00000000	-1.00e+00	9.00e+00	1.11e-01
2	1.11111111	-5.75e-02	8.00e+00	7.19e-03
3	1.11829936	-2.08e-04	7.94e+00	2.62e-05
4	1.11832559	-2.75e-09	7.94e+00	3.47e-10

>>> CONVERGED: Root = 1.11832559 after 4 iterations
 Final function value: f(x) = 6.66e-16

5.5 Convergence table Bisection $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

CONVERGENCE TABLE: $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

Interval: [4, 5], tol=1e-6

Iter	a	b	Middle	f(Middle)	Interval Size
1	4.000000	5.000000	4.500000	-2.42e+00	1.00e+00
2	4.000000	4.500000	4.250000	-4.87e-01	5.00e-01
3	4.000000	4.250000	4.125000	6.41e-01	2.50e-01
4	4.125000	4.250000	4.187500	6.70e-02	1.25e-01
5	4.187500	4.250000	4.218750	-2.13e-01	6.25e-02
6	4.187500	4.218750	4.203125	-7.35e-02	3.12e-02
7	4.187500	4.203125	4.195312	-3.43e-03	1.56e-02
8	4.187500	4.195312	4.191406	3.17e-02	7.81e-03
9	4.191406	4.195312	4.193359	1.41e-02	3.91e-03
10	4.193359	4.195312	4.194336	5.36e-03	1.95e-03
11	4.194336	4.195312	4.194824	9.66e-04	9.77e-04
12	4.194824	4.195312	4.195068	-1.23e-03	4.88e-04
13	4.194824	4.195068	4.194946	-1.32e-04	2.44e-04
14	4.194824	4.194946	4.194885	4.17e-04	1.22e-04
15	4.194885	4.194946	4.194916	1.42e-04	6.10e-05
16	4.194916	4.194946	4.194931	4.88e-06	3.05e-05
17	4.194931	4.194946	4.194939	-6.38e-05	1.53e-05
18	4.194931	4.194939	4.194935	-2.94e-05	7.63e-06
19	4.194931	4.194935	4.194933	-1.23e-05	3.81e-06
20	4.194931	4.194933	4.194932	-3.70e-06	1.91e-06

>>> CONVERGED: Root = 4.19493151 after 20 iterations

5.6 Convergence table Newton $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

CONVERGENCE TABLE: $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

Initial guess: x0=4, tol=1e-6

Iter	x _n	f(x _n)	f'(x _n)	dx
-----	-----	-----	-----	-----

1		4.00000000		1.83e+00		-9.69e+00		1.89e-01
2		4.18910281		5.25e-02		-9.03e+00		5.82e-03
3		4.19492186		8.74e-05		-9.00e+00		9.71e-06
4		4.19493157		2.46e-10		-9.00e+00		2.74e-11

```
>>> CONVERGED: Root = 4.19493157 after 4 iterations
Final function value: f(x) = 6.66e-16
```

5.7 Your code and functions

```
% assignment1_task2_master.m
clear; close all; clc;

%% PARAMETERS
tol_bisect = 1e-6;
imax_bisect = 50;

tol_newton = 1e-6;
imax_newton = 20;

%% -----
% FIGURE 1.1 – Bisection Method, Function 1
% f(x) = sin(x) * sinh(x)
% interval [1,4] (contains pi)
%% -----
fprintf('\n==== FIGURE 1.1: Bisection for f1(x)=sin(x)*sinh(x) ==== \n');

a1 = 3; b1 = 4;
root_f1 = call_bisection('func1', a1, b1, tol_bisect, imax_bisect);

% print root
fprintf('Final root (func1) = %.12f \n', root_f1);

% plot
x1 = linspace(a1,b1,500);
y1 = func1(x1);
figure(1)
plot(x1,y1,'b-','LineWidth',2); hold on
plot(root_f1, func1(root_f1),'ro','MarkerFaceColor','r','MarkerSize',8)
grid on
title(sprintf('Figure 1.1 – f(x)=sin(x) sinh(x) | Root: x = %.6f', root_f1))
xlabel('x'); ylabel('f(x)')
legend('f(x)=sin(x)sinh(x)', sprintf('Root: x = %.6f', root_f1),
'Location','best')

%% -----
% FIGURE 1.2 – Bisection Method, Function 2
% f(x) = sqrt(R^2 - x^2) - x*tan(x), R = 10
% interval [4, 4.7]
%% -----
fprintf('\n==== FIGURE 1.2: Bisection for f2(x)=sqrt(R^2-x^2)-x tan(x) (R=10)
==== \n');

a2 = 4; b2 = 4.7;
```

```

root_f2 = call_bisection('func2', a2, b2, tol_bisect, imax_bisect);

fprintf('Final root (func2) = %.12f\n', root_f2);

% plot
x2 = linspace(a2,b2,500);
y2 = func2(x2);
figure(2)
plot(x2,y2,'k-','LineWidth',2); hold on
plot(root_f2, func2(root_f2),'ro','MarkerFaceColor','r','MarkerSize',8)
grid on
title(sprintf('Figure 1.2 – f(x)=sqrt(R^2-x^2)-x tan(x) | Root: x = %.6f',
root_f2))
xlabel('x'); ylabel('f(x)')
legend('f(x)=sqrt(R^2-x^2)-x tan(x)', sprintf('Root: x = %.6f', root_f2),
'Location','best')

%% -----
% FIGURE 2.1 – Newton Method (Task 2b)
% f(x) = exp(x) - (1 + x + x^2/2), x0 = 5
%% -----
fprintf('\n=== FIGURE 2.1: Newton for f(x)=exp(x)-(1+x+x^2/2) ===\n');

x0_exp = 5;
% newton prints iterations as it runs (see newton.m)
root_N1 = call_newton('func_exp','dfunc_exp', x0_exp, tol_newton, imax_newton);

fprintf('Final Newton root (func_exp) = %.12f\n', root_N1);

% plot
xx = linspace(-1,6,500);
yy = func_exp(xx);
figure(3)
plot(xx,yy,'b-','LineWidth',2); hold on
plot(root_N1, func_exp(root_N1),'ro','MarkerFaceColor','r','MarkerSize',8)
grid on
title(sprintf('Figure 2.1 – Newton: f(x)=e^x-(1+x+x^2/2) | Root: x = %.6f',
root_N1))
xlabel('x'); ylabel('f(x)')
legend('f(x)=e^x-(1+x+x^2/2)', sprintf('Root: x = %.6f', root_N1),
'Location','best')

%% -----
% FIGURE 2.2 – Newton Method (Task 2c)
% f(x) = 10*ln(x) - x, x0 = 1
%% -----
fprintf('\n=== FIGURE 2.2: Newton for f(x)=10 ln(x) - x ===\n');

x0_log = 1;
root_N2 = call_newton('func_log','dfunc_log', x0_log, tol_newton, imax_newton);

fprintf('Final Newton root (func_log) = %.12f\n', root_N2);

% plot
xx = linspace(0.1,20,500);
yy = func_log(xx);
figure(4)

```

```

plot(xx,yy,'m-','LineWidth',2); hold on
plot(root_N2, func_log(root_N2),'ro','MarkerFaceColor','r','MarkerSize',8)
grid on
title(sprintf('Figure 2.2 – Newton: f(x)=10 ln(x)-x | Root: x = %.6f', root_N2))
xlabel('x'); ylabel('f(x)')
legend('f(x)=10 ln(x)-x', sprintf('Root: x = %.6f', root_N2), 'Location','best')

%% -----
% FIGURE 3.1 – Intersection Plot (visual)
% functions: y1 = 100 e^{-x}, y2 = 5 sin(pi x /2)
% find root near x0 = 4 using Newton with tol = 1e-10
%% -----
fprintf('\n=== FIGURE 3.1: Intersection plot & Newton root near x=4 ===\n');

x0_int = 4;
tol_int = 1e-10;
imax_int = 50;

root_inter = call_newton('func_intersect','dfunc_intersect', x0_int, tol_int,
imax_int);

fprintf('Final Newton root (intersection near 4) = %.12f\n', root_inter);

x_plot = linspace(3,10,800);
y_left = 100*exp(-x_plot);
y_right = 5*sin(pi*x_plot/2);

figure(5)
plot(x_plot,y_left,'r-','LineWidth',2); hold on
plot(x_plot,y_right,'b-','LineWidth',2);
plot(root_inter, 100*exp(-root_inter),'ko','MarkerFaceColor','g','MarkerSize',8)
grid on
title(sprintf('Figure 3.1 – Intersection: root near 4 = %.6f', root_inter))
xlabel('x'); ylabel('y')
legend('100 e^{-x}','5 sin(\pi x/2)', sprintf('Root: %.6f',
root_inter),'Location','best')

%% -----
% FIGURE 4.1 & 4.2 – Bracketing (multiple roots) then refine with Bisection &
Newton
% interval [4,10]
%% -----
fprintf('\n=== FIGURE 4: Bracketing and refining multiple roots for intersection
===\n');

a = 4; b = 10;
n_samples = 200; % initial samples used in bracket_roots
brackets = bracket_roots(@func_intersect, a, b, n_samples);

if isempty(brackets)
    warning('No sign-change brackets found in [%g,%g].', a, b);
else
    fprintf('Brackets found (approx):\n');
    disp(brackets);
end

nB = size(brackets,1);

```

```

roots_bisect = zeros(nB,1);
roots_newton = zeros(nB,1);

% refine
for k = 1:nB
    A = brackets(k,1);
    B = brackets(k,2);
    fprintf('\n--- Root region %d: [%.6f, %.6f] ---\n', k, A, B);

    % Bisection
    roots_bisect(k) = call_bisection('func_intersect', A, B, tol_bisect,
imax_bisect);
    fprintf('Bisect result: x = %.12f\n', roots_bisect(k));

    % Newton (starting from midpoint)
    mid = (A + B)/2;
    roots_newton(k) = call_newton('func_intersect','dfunc_intersect', mid,
tol_bisect, imax_newton);
    fprintf('Newton result: x = %.12f\n', roots_newton(k));
end

% plot bisection roots (Figure 4.1)
figure(6)
plot(x_plot,y_left,'r-','LineWidth',2); hold on
plot(x_plot,y_right,'b-','LineWidth',2);
for r = roots_bisect.'
    plot(r,100*exp(-r),'ko','MarkerFaceColor','g','MarkerSize',6)
end
grid on
title('Figure 4.1 – Multiple roots (Bisection refined)')
xlabel('x'); ylabel('y')
legend('100 e^{-x}','5 sin(\pi x/2)','Roots (bisection)','Location','best')

% plot newton roots (Figure 4.2)
figure(7)
plot(x_plot,y_left,'r-','LineWidth',2); hold on
plot(x_plot,y_right,'b-','LineWidth',2);
for r = roots_newton.'
    plot(r,100*exp(-r),'ko','MarkerFaceColor','g','MarkerSize',6)
end
grid on
title('Figure 4.2 – Multiple roots (Newton refined)')
xlabel('x'); ylabel('y')
legend('100 e^{-x}','5 sin(\pi x/2)','Roots (Newton)','Location','best')

%% -----
% Final summary output
%% -----
fprintf('\n==== FINAL ROOT SUMMARY ==== \n');
fprintf('Figure 1.1 (func1) root:      x = %.12f\n', root_f1);
fprintf('Figure 1.2 (func2) root:      x = %.12f\n', root_f2);
fprintf('Figure 2.1 (func_exp) root:    x = %.12f\n', root_N1);
fprintf('Figure 2.2 (func_log) root:    x = %.12f\n', root_N2);
fprintf('Figure 3.1 (intersection) root:x = %.12f\n', root_inter);

if nB>0
    fprintf('Figure 4.1 roots (bisection):\n');
    disp(roots_bisect);
    fprintf('Figure 4.2 roots (newton):\n');

```

Module code

Task 2

Name

```
        disp(roots_newton);  
end  
fprintf('==== END ==== \n');
```